Image Processing

Cosimo Distante

Lecture 6: Monochrome and Color processing
**Pointwise operator**: algorithms that execute simple operation on the single pixel without involving neighboring pixels

\[ I_0(i,j) = O_{\text{pointwise}}[I_1(i,j)] \]

If \( I_1(i,j) > 150 \) then \( I_0(i,j) = 1 \)
else \( I_0(i,j) = 0 \)
**Local Operator:** algorithms that define the new value of a pixel based on the intensity values of the neighboring pixels

$I_i$ input image,
$F(i,j)$ a window defined over the analysing pixel

$I_0$ output image

$$I_0(i,j) = \text{O}_{\text{locale}} \{ I_i(i_k,j_l); (i_k,j_l) \in F(i,j) \}$$

Median filter with window size $3 \times 3$
Global Operator: algorithms that extract global information from the image
They use all pixels of the image

\[ R = O_{\text{global}}[I_I(i,j)] \]
Histogram

$H(x)$ is the frequency of intensity value $x$

The histogram $H_I(x)$ can be seen as the results of the global operator for the input image $I$.

With the histogram we lose spatial information.
Contrast manipulation

Given:
x gray level of input image $I_1(i,j)$,
y gray level of output image $I_0(i,j)$,
$T(x)$ the gray level transform (manipulation)

\[ y = T(x) \]
Linear

\[ y = \beta(x - x_a) \]

with \( x_a \leq x \leq x_b \) and \( \beta = \frac{\Delta y}{\Delta x} \) the stretching coefficient obtained by the ratio of the output gray level range and input gray level range \( \Delta x = x_b - x_a \).

Example: the interval of the most frequent levels in the image is \([50..100]\)
Then \( \Delta x = 50 \) different levels
That can be stretched to 256 levels (\(\Delta y\)) to have a better quality.
Linear stretching
Linear with multiple paths

\[ y = \begin{cases} 
\alpha x & 0 \leq x < x_a \\
\beta(x - x_a) + y_a & x_a \leq x < x_b \\
\delta(x - x_b) + y_b & x_b \leq x < L_{\text{max}} 
\end{cases} \]

(1.9)

\[ \alpha = \frac{y_a}{x_a}, \quad \beta = \frac{y_b - y_a}{x_b - x_a}, \quad \delta = \frac{L_{\text{max}}(y - y_b)}{L_{\text{max}}(x - x_b)} \]

\( y_a \) and \( y_b \) are constants used to increase or decrease the global illumination
Linear with multiple paths – special case

If $\alpha=0$ and $\delta=0$
Then stretching is related to interval $(x_a,x_b)$
While are excluded (clipping) gray levels less than $x_a$ and greater than $x_b$

This is useful when we know the object is in $(x_a,x_b)$ in order to **segment it**.
IF $x_a=x_b=x_T$, the output image is named **Binary Image**.
Non Linear

**Quadratic transformation:**

\[ y = x^2 \]

Expand high gray level and compress lower gray level

**Square root transf.**

\[ y = \sqrt{x} \]

Opposite behavior as previous transf.

**Log transform**

\[ y = \frac{\log_e (1 + x)}{\log_e [1 + \max(x)]} \]

Applied when the range of gray levels in input image is much wider than the wanted range in output image (in Fourier representation)
Negative

Complementing with respect to the maximum gray level of the input image

Inverse

\[ y = L \max - x \]

\[ y = \frac{1}{x} \quad \text{with } x > 0 \]

Useful to visualize very dark details of an image
Histogram Equalization

• The histogram equalization transformation generates an image with equally likely intensity values
• The intensity values in the output image cover the full range, [0 1]
• The resulting image has higher dynamic range
• Recall the values in the normalized histogram are approximately the probability of occurrence of those values
• The histogram equalization transform is the cumulative distribution function (CDF)
CUMULATIVE DISTRIBUTION FUNCTION

Histogram

CDF
Histogram equalization

\[ H_O(y_k) = \sum_{i=0}^{L_{\text{max}}} \frac{H_I(x_i)}{L_{\text{max}}} \]  \quad \text{for} \quad i = 0, \ldots, L_{\text{max}} \quad \text{and} \quad \forall k \in [1, N_{L_{\text{max}}}] \]
Histogram equalization
Histogram equalization Algorithm

Given NxM tha image size and $L_{\text{max}}$ the maximum gray levels

First step is to create the histogram of the image $H_I$

3: for $i = 0, \ldots N - 1$ do
4: for $j = 0, \ldots M - 1$ do
5: $x \leftarrow I_I(i, j)$
6: $H_I(x) \leftarrow H_I(x) + 1$
7: end for
8: end for

Then build the cumulative histogram $H_c$

9: $H_c(0) \leftarrow H_I(0)$
10: for $k = 1, \ldots, L_{\text{max}} - 1$ do
11: $H_c(k) \leftarrow H_c(k - 1) + H_I(k)$
12: end for

Then compute the mapping function

$y = T(x) = \text{round} \left( \frac{L_{\text{max}} - 1}{N \times M} H_c(x) \right) \quad 0 \leq x < L_{\text{max}}$

and remap every pixel

14: for $i = 0, \ldots N - 1$ do
15: for $j = 0, \ldots M - 1$ do
16: $I_O(i, j) = T(I_I(i, j))$
17: end for
18: end for
Histogram Equalization

Input Image

Output Image
Histogram Equalization Example

- \( g = \text{histeq}(f, n\text{lev}) \) where \( f \) is the original image and \( n\text{lev} \) number of intensity levels in output image
Adaptive Histogram Equalization (AHE)

Since our eyes adapt to local regions instead of entire image, it is useful to optimize image enhancement locally.

The image is divided in a grid of non-overlapping regions.

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Contrast Limited Adaptive Histogram Equalization (CLAHE)

Operate a clip on the equalized histogram

Compute the cumulative histogram

\[ f_{i,j}(n) = \frac{(N - 1)}{M} \cdot \sum_{k=0}^{n} h_{i,j}(k); \]

\[ n = 1, 2, 3 \ldots, N - 1 \]

\( \beta \) is the clip limit and \( \alpha \) clip factor

\[ \beta = \frac{M}{N} \left(1 + \frac{\alpha}{100}(s_{\text{max}} - 1)\right) \]

\( s_{\text{max}} \) maximum allowed slope

For X-ray images \( s_{\text{max}} = 4 \)

Perform histogram equalization locally and operate a clipping redistributing the pixels

Notation change

N is the number of gray levels
M the total number of pixels in Image
Contrast Limited Adaptive Histogram Equalization (CLAHE)

\[
\text{Excess} = 0
\]
\[
\text{For } n = 0,1,2,\ldots,N-1
\]
\[
\quad \text{If } h(n) > \beta, \text{ Then}
\]
\[
\quad \quad \text{Excess} \leftarrow \text{Excess} + h(n) - \beta
\]
\[
\quad \quad h(n) \leftarrow \beta
\]
\[
\quad \text{End If}
\]
\[
\text{End For}
\]
\[
\text{m} = \frac{\text{Excess}}{N}
\]
\[
\text{For } n = 0,1,2,\ldots,N-1
\]
\[
\quad \text{If } h(n) < \beta - m, \text{ Then}
\]
\[
\quad \quad h(n) \leftarrow h(n) + m
\]
\[
\quad \quad \text{Excess} \leftarrow \text{Excess} - m
\]
\[
\quad \text{Else If } h(n) < \beta, \text{ Then}
\]
\[
\quad \quad \text{Excess} \leftarrow \text{Excess} - \beta + h(n)
\]
\[
\quad \quad h(n) \leftarrow \beta
\]
\[
\quad \text{End If}
\]
\[
\text{End For}
\]
\[
\text{While } \text{Excess} > 0
\]
\[
\quad \text{For } n = 0,1,2,\ldots,N-1
\]
\[
\quad \quad \text{If Excess} > 0, \text{ Then}
\]
\[
\quad \quad \quad \text{If } h(n) < \beta, \text{ Then}
\]
\[
\quad \quad \quad \quad h(n) \leftarrow h(n) + 1
\]
\[
\quad \quad \quad \quad \text{Excess} \leftarrow \text{Excess} - 1
\]
\[
\quad \quad \text{End If}
\]
\[
\quad \text{End If}
\]
\[
\text{End For}
\]
\[
\text{End While}
\]

**Notation change**

N is the number of gray levels

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Contrast Limited Adaptive Histogram Equalization (CLAHE)